

Contents

Preface to the Series, v

Preface To This Volume, vii

Organization of the Book, vii

Contents, xi

Chapter 1: Introduction, 1

What is Abstract Algebra?, 1

Chapter 2: Groups, 13

Algebraic Operations and Closure, 13

The Definition of a Group, 14

Examples of Groups, 18

The Cyclic Groups, 24

The Order of a Group Element, 28

The Exponents of a Group, 32

Subgroups of a Group, 34

The Center of a Group, 36

Group Homomorphisms, 37

The Classification Problem of Group Theory, 41

Conjugacy, 44

Exercises, 46

Chapter 3: Symmetric Groups and Free Groups, 51

The Symmetric Groups, 51

Cayley's Theorem, 59

An Historical Perspective: Galois Tables, 60

The Free Groups, 62

Exercises, 69

Chapter 4: Applications of Groups in Mathematics, 73

Groups In Geometry, 73

Groups In Analysis, 73

Groups In Algebraic Topology, 74

Groups In Number Theory, 75

Groups in Coding Theory, 77

Chapter 5: Subgroups and External Direct Products, 83

Groups In Cryptology, 80

The Subgroup Lattice of a Group, 83

The Subgroup Generated by a Subset, 84

The Set Product of Subgroups, 87

Cosets and Lagrange's Theorem, 89
The External Direct Product of Groups, 92
Exercises, 96

Chapter 6: Finitely-Generated Groups; Group Templates, 101

Finitely-Generated Groups, 101
Defining a Group, 107
Group Templates, 108
Exercises, 118

Chapter 7: Special Families of Groups, 121

More on Cyclic Groups, 121
The Euclidean Group, 129
The Dihedral Groups, 131
The Additive Rationals, 135

Chapter 8: Cosets, Index and Normal Subgroups, 141

Exercises, 138
Cosets and Index, 141
Normal Subgroups and Quotient Groups, 142
The Normal Closure, the Normal Interior and the Normalizer, 152
Gigantic Subgroups of a Finite Group, 154
The Dicyclic Groups, 157
Counting Conjugates, 159
Exercises, 160

Chapter 9: Direct Products, Cauchy's Theorem and Finite Abelian Groups, 165

Direct Products, 165
Cauchy's Theorem, 174
Finite Abelian Groups, 176

Chapter 10: The Isomorphism Theorems, 183

Exercises, 181
Homomorphisms, 183
Kernels and the Natural Projection, 184
The Isomorphism Theorems, 186
The Correspondence Theorem, 188
Exercises, 191

Chapter 11: Symmetric Groups, 195

The Definition and Cycle Decomposition, 195
Parity, 196
Generating Sets for S_n and A_n , 198
Subgroups of S_n and A_n , 199
Conjugation in the Alternating Group, 199
The Simplicity of A_n , 203
Normal Subgroups of S_n , 206
Exercises, 207

Chapter 12: Group Actions, 211

Representations, 211
Group Actions, 211
Translation by a on G , 217
Conjugation by a on the Conjugates of a Subgroup, 218
Conjugation by a on G , 219
Transitive Group Actions, 220

Exercises, 220

Chapter 13: Sylow Theory, 225

Sylow Subgroups, 225

Normal Sylow Subgroups, 226

The Sylow Theorems, 226

The Search for Normal Subgroups, 229

Groups of Order, 232

Groups of Small Order, 236

Exercises, 239

Appendix A: Background, 241

Multisets, 241

Cartesian Products, 242

Binary Relations, 244

Partitions and Equivalence Relations, 244

Appendix B: Partially Ordered Sets, 247

Partially Ordered Sets, 247

Zorn's Lemma, 251

Lattices, 251

Intersection Structures, 254

Final Remarks, 256

Exercises, 257

Solutions to Selected Exercises, 259

References, 309

Index of Symbols, 311

Books on Group Theory, 309

Papers on Group Theory, 309

References on the Burnside Problem, 309

Books By This Author, 310

Index, 313